

# *Hydrobiological Modelling*

**Processes, numerical methods and applications**

*by*

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**All units are SI, unless otherwise noted.**

The cover photograph is a view of Lake Macquarie, NSW, Australia, looking south from the Sea Eagle lookout in Green Point Park. It was taken with a 7 Megapixel camera and stored as a 2 Megabyte JPEG.

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